



# A prediction for $|\mathcal{U}_{e3}|$ from patterns in the charged lepton spectra

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## Abstract

It is shown that empirical relations between the charged lepton spectra and the quark spectra together with a bimaximal or near bimaximal neutrino mixing matrix necessarily imply that there is a contribution to  $|\mathcal{U}_{e3}|$  given by  $\theta_C/3\sqrt{2} \approx \sqrt{m_e/2m_\mu} \approx 0.052$ , where  $\theta_C$  is the Cabibbo angle. This prediction could be tested in the near future reactor experiments. The charged lepton mixing also generates a less robust prediction for the angle  $\theta_{23}$  and a small contribution to the phase  $\delta$ .

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## 1. Introduction

During the last year our knowledge of the leptonic mixing matrix has reached the precision level. The most recent 90% C.L. experimental results [1–3] and several global fits [4–7] have improved our knowledge of the neutrino mass differences and indicate that the atmospheric mixing is almost maximal while the solar mixing deviates from maximality in a particular way. In the standard notation,

$$\sin \theta_{12} = 0.53 \pm 0.04, \quad (1)$$

$$\sin \theta_{23} = 0.70 \pm 0.11, \quad (2)$$

$$\sin \theta_{13} < 0.15, \quad (3)$$

$$\Delta m_{\text{sun}}^2 = \Delta m_{21}^2 = (8.2 \pm 0.6) \times 10^{-5} \text{ eV}^2, \quad (4)$$

$$\Delta m_{\text{atm}}^2 = \Delta m_{23}^2 = (2.45 \pm 0.55) \times 10^{-3} \text{ eV}^2. \quad (5)$$

This substantial improvement has confirmed that the leptonic mixing matrix, called MNSP matrix, is nearly bimaximal [8,9] and the particular deviation from bimaximality observed has revealed a surprising relation between the Cabibbo angle,  $\theta_C$  and the solar mixing angle [10] sometimes called the quark–lepton complementarity relation,  $\theta_C + \theta_{12} \approx \pi/4$ , hereafter referred to as QLC relation. Therefore, based on the experimental data it is convenient to define the following

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parametrization [11] of the MNSP matrix,

$$\mathcal{V}_{\text{MNSP}} = \begin{bmatrix} \frac{1}{\sqrt{2}}(1+\lambda) & -\frac{1}{\sqrt{2}}(1-\lambda) & 0 \\ \frac{1}{2}(1-\lambda) & \frac{1}{2}(1+\lambda) & -\frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-\lambda) & \frac{1}{2}(1+\lambda) & \frac{1}{\sqrt{2}} \end{bmatrix} + \mathcal{O}(\lambda^2), \quad (6)$$

where we will use  $\lambda$  alternatively to refer to the Cabibbo angle. We note that the mixing angle  $\theta_{13}$  is at present constrained to be  $\theta_{13} < 0.15 \simeq 3\lambda^2$  by the non-observation of neutrino oscillations at the CHOOZ experiment [3] and a fit to the global data [7]. There will be many efforts in the near future to measure all parameters in the neutrino mixing matrix to a much higher degree of precision. In the immediate future, reactor neutrino experiments will strengthen the bounds on  $\theta_{13}$  or will actually measure a non-zero value for it. Is it possible that  $\theta_{13}$  is actually zero or should we expect that future experiments will measure a non-zero value? Various symmetry schemes have been proposed in the literature that lead to a prediction for  $\theta_{13}$  while generating a near bimaximal MNSP matrix [12]. On the other hand, it is known that the MNSP mixing matrix in a general leptonic basis receives contributions from both the neutrino and the charged lepton mass matrices,

$$\mathcal{V}_{\text{MNSP}} = (\mathcal{V}_L^l)^\dagger \mathcal{V}_\nu, \quad (7)$$

where  $\mathcal{V}_\nu$  is the neutrino diagonalization matrix and  $\mathcal{V}_L^l$  is the left-handed charged lepton diagonalization matrix,  $\mathcal{M}_l^{\text{diag}} = (\mathcal{V}_L^l)^\dagger \mathcal{M}_l \mathcal{V}_R^l$ . It is the main purpose of this Letter to show that, irrespective of what is the precise nature of the underlying symmetry that determines the exact deviation from bimaximality in the neutrino mass matrix, the existence of precise empirical relations between the charged lepton spectra, the quark spectra and the Cabibbo angle indicates that the associated charged lepton mixing together with a near bimaximal neutrino mixing matrix must generate a contribution to  $|\mathcal{U}_{e3}|$ . It is plausible that this contribution is the dominant source of  $|\mathcal{U}_{e3}|$ . Here  $\mathcal{U}_{aj}$  ( $a = e, \mu, \tau$  and  $j = 1, 2, 3$ ) denote the elements of the MNSP matrix.

## 2. A prediction for $|\mathcal{U}_{e3}|$

There is an empirical relation which has been known for quite a long time [13,14],

$$|V_{us}| \approx \left[ \frac{m_d}{m_s} \right]^{1/2} \approx 3 \left[ \frac{m_e}{m_\mu} \right]^{1/2}. \quad (8)$$

This relation has been recently analyzed with precision by one of the authors who noted that indeed the relation surprisingly works at the level of  $\pm 16\%$ , as the following ratio shows (see Ref. [15] for details),

$$\frac{[m_d/m_s]^{1/2}}{[m_e/m_\mu]^{1/2}} = 3.06 \pm 0.48. \quad (9)$$

The relation between the Cabibbo angle and the down-strange quark mass ratio can be simply explained, as known from the 70s [16], if the down quark mass is generated from the mixing between the first and second families. In this case, one expects that there is a quark basis where the normalized down-type quark mass matrix is given to leading order by

$$\hat{\mathcal{M}}_D = \begin{bmatrix} 0 & \left(\frac{m_s m_d}{m_b^2}\right)^{1/2} & \mathcal{O}(\lambda^3) \\ \left(\frac{m_s m_d}{m_b^2}\right)^{1/2} & \left(\frac{m_s}{m_b}\right) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{bmatrix}. \quad (10)$$

The order of magnitude in the coefficients  $(\hat{\mathcal{M}}_D)_{13}$  and  $(\hat{\mathcal{M}}_D)_{23}$  is obtained by requiring these entries not to affect the quark mass ratios predicted by the matrix to leading order. Analogously, the relation between the Cabibbo angle and the electron–muon mass ratio can also be simply explained if the electron mass is generated from the mixing between the first and second lepton families. This implies that there is a leptonic basis where the charged lepton mass matrix is given to leading order by

$$\hat{\mathcal{M}}_l = \begin{bmatrix} 0 & \left(\frac{m_\mu m_e}{m_\tau^2}\right)^{1/2} & \mathcal{O}(\lambda^3) \\ \left(\frac{m_\mu m_e}{m_\tau^2}\right)^{1/2} & \left(\frac{m_\mu}{m_\tau}\right) & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{bmatrix}. \quad (11)$$

The order of magnitude in the coefficients  $(\hat{\mathcal{M}}_l)_{13}$  and  $(\hat{\mathcal{M}}_l)_{23}$  can be obtained by requiring these entries to not modify the leading order terms for the charged lepton mass ratios. Such a form for the charged lepton mass matrix is also obtained in the mass matrix ansatz in Ref. [17]. From the matrix in Eq. (11) and the empirical relation in Eq. (8) follows that the charged lepton mixing matrix is given in this leptonic basis by

$$\mathcal{V}_L^l = \begin{bmatrix} 1 & \lambda/3 & \mathcal{O}(\lambda^3) \\ \lambda/3 & 1 & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{bmatrix}. \quad (12)$$

It is known that the relation in Eq. (8) between the quark masses, the charged lepton masses and the Cabibbo angle could be explained in some GUT models [14]. In that case it is plausible that the basis where  $\mathcal{V}_L^l$  adopts the form given by Eq. (12) while the down-type quark mass matrix adopts the form given by Eq. (10) is the gauge flavor basis of the GUT model where quark and leptons unify in the same representations. Let us assume that the charged lepton mixing matrix is given to leading order in  $\lambda$  by Eq. (12) and in the same basis the underlying neutrino mass matrix generates an exactly bimaximal neutrino mixing matrix,

$$\mathcal{V}_\nu = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}. \quad (13)$$

In this case one expects the charged lepton mixing to induce a non-zero  $|\mathcal{U}_{e3}|$  [18]. In our case, as long as the mixing in the neutrino sector is approximately bimaximal, we find

$$|\mathcal{U}_{e3}| = \frac{\lambda}{3\sqrt{2}} = \left(\frac{m_e}{2m_\mu}\right)^{1/2} \approx 0.052 \pm 0.001. \quad (14)$$

The present fit to the global data indicates that  $\sin\theta_{13} < 0.15$  at 90% C.L. [7]. There are some reactor experiments proposed for the future: BRAIDWOOD in Illinois, DAYA BAY in China and KASKA in Japan, that are expected to reach the level of  $\sin\theta_{13} \approx 0.05$  [19]. Since CHOOZ II will only reach a sensitivity in  $\sin\theta_{13}$  of  $\approx 0.08$  at 90% C.L. after 3 years of operation [20], we expect it to obtain a null result. It has been estimated that neutrino factories will reach values of the order  $|\mathcal{U}_{e3}| \approx 0.025$  [21]. This prediction is rather robust as it will follow even if the neutrino mixing matrix is not bimaximal, but merely if the third column has the form as in Eq. (13). For example, a neutrino mixing matrix of the so-called tri-bimaximal [22] form will also yield the same result. We have learnt during the elaboration of this Letter of a simultaneous derivation of this prediction by Bjorken [23] in the context of the model proposed in Ref. [17].

### 3. A prediction for $\sin\theta_{23}$

In this section we would like to point out that based on a second empirical relation between the fermion

masses and the CKM elements recently unveiled [15, 24] it is plausible to expect also a contribution to  $\sin(\theta_{23})$ , coming from the (23) mixing in the charged lepton mass matrix, and a non-zero CP-violating phase in the MNSP matrix. Nevertheless, the predictions in this section for  $\theta_{23}$  and  $\delta$  are less robust than the one for  $\mathcal{U}_{e3}$ . The new empirical relation mentioned above is given by

$$\theta \approx \left[\frac{m_s^3}{m_b^2 m_d}\right]^{1/2} \approx \left[\frac{m_c^3}{m_t^2 m_u}\right]^{1/2} \approx \frac{1}{9} \left[\frac{m_\mu^3}{m_\tau^2 m_e}\right]^{1/2}. \quad (15)$$

This relation together with an additional empirical relation with the quark mixing angles,

$$\theta \approx \frac{1}{2} \left| \frac{V_{cb}}{V_{us}} \right| = 0.093 \pm 0.003, \quad (16)$$

implies [15] that the quark mass matrices can be reconstructed to leading order as a function of the two basic flavor parameters:  $\theta$  and  $\lambda$ . In certain basis where the up-type quark mass matrix is diagonal the reconstructed normalized down-type quark matrix would be given by

$$\hat{\mathcal{M}}_d = \begin{bmatrix} 0 & \theta\lambda^2 & \theta\lambda^2 e^{-i\gamma} \\ \theta\lambda^2 & \theta\lambda & 2\theta\lambda \\ \theta\lambda^2 e^{i\gamma} & 2\theta\lambda & 1 \end{bmatrix}, \quad (17)$$

where  $\gamma$  is the standard CP-violating phase. Here  $\hat{\mathcal{M}}_d$  is bidiagonalized by  $(\mathcal{V}_L^d)^\dagger \hat{\mathcal{M}}_d \mathcal{V}_R^d$ . In this quark basis,  $\mathcal{V}_{\text{CKM}} = \mathcal{V}_L^d$ , which to leading order is

$$\mathcal{V}_{\text{CKM}} = \begin{bmatrix} 1 - \lambda^2/2 & \lambda & -\theta\lambda^2 e^{-i\gamma} \\ -\lambda & 1 - \lambda^2/2 & -2\theta\lambda \\ (e^{i\gamma} - 2)\theta\lambda^2 & 2\theta\lambda & 1 - 2\theta^2\lambda^2 \end{bmatrix}. \quad (18)$$

It has been shown [15] that this simple mass matrix  $\hat{\mathcal{M}}_d$  fits all the experimental data with precision and additionally predicts a simple successful relation between the quark CP phases,  $\beta = \text{Arg}[2 - e^{-i\gamma}]$ . If there is a connection between the charged lepton mass matrix and the down-type quark matrix, as is the case in some GUT models,<sup>1</sup> we expect that there is a leptonic basis where the normalized charged lepton mass

<sup>1</sup> For instance, in SU(5) models where the Higgs field giving mass to the charged leptons and down-type quarks transforms under the representation **45** of SU(5) [14].

matrix is given by

$$\hat{\mathcal{M}}_l = \begin{bmatrix} 0 & \theta\lambda^2 & \theta\lambda^2 e^{-i\gamma} \\ \theta\lambda^2 & 3\theta\lambda & 2\theta\lambda \\ \theta\lambda^2 e^{i\gamma} & 2\theta\lambda & 1 \end{bmatrix}. \quad (19)$$

In this basis the charged lepton mixing matrix would be given to leading order by

$$\mathcal{V}_L^l = \begin{bmatrix} 1 & \lambda/3 & -\theta\lambda^2 e^{-i\gamma} \\ -\lambda/3 & 1 & -2\theta\lambda \\ (e^{i\gamma} - \frac{2}{3})\theta\lambda^2 & 2\theta\lambda & 1 \end{bmatrix}, \quad (20)$$

where the deviation from unitarity is at most of order  $\theta\lambda^3$ . If we assume that the neutrino mixing matrix is exactly bimaximal (or rather the third column has that form), we obtain, using Eq. (7), a prediction for  $\sin^2(2\theta_{23})$  given by

$$\sin^2(2\theta_{23}) = 4|\mathcal{U}_{\mu 3}\mathcal{U}_{\tau 3}|^2 = 1 - \frac{4}{9}\left(\frac{m_\mu}{m_\tau}\right)^2. \quad (21)$$

This corresponds to  $\sin^2(2\theta_{23}) \approx 0.998$ , or that  $\theta_{23}$  differs from  $\pi/4$  by  $\simeq 1.7^\circ$ . We expect that future experiments could rule out this prediction for  $\theta_{23}$ . The magnitude of the CP-violating effects in neutrino oscillations is controlled by the rephasing-invariant  $J_{\text{CP}}^\nu = \text{Im}[\mathcal{U}_{km}^* \mathcal{U}_{lm} \mathcal{U}_{kn} \mathcal{U}_{ln}^*]$  (irrespective of the indices). If the neutrino mixing matrix was nearly bimaximal and CP conserving the source of the CP-violating phase in the MNSP matrix would arise from the phase present in the charged lepton mixing matrix,<sup>2</sup> which based on the matrix in Eq. (20) is given by

$$J_{\text{CP}}^\nu = -\frac{\theta\lambda^2}{4\sqrt{2}} \sin \gamma. \quad (22)$$

The phase  $\delta$  would be given in this case by  $\tan \delta = 3\lambda\theta \sin \gamma$ . Experimentally  $\gamma$  seems to be a large angle [25]. The 2004 winter global fit of the CKM elements obtained using the program CKMFitter [26] gives us  $\gamma_{\text{exp}} = 61^\circ \pm 11^\circ$ . Therefore,  $J_{\text{CP}}^\nu$  could be as large as about  $10^{-3}$ , which corresponds to a phase  $\delta \approx 3^\circ$ . Nevertheless, the CP-conservation of the neutrino mixing matrix is a very strong assumption. It is known that if neutrinos are Majorana particles some neutrino phases cannot be absorbed by redefinition of

the neutrino fields [27] and in general there would be a contribution to  $J_{\text{CP}}^\nu$  given by

$$J_{\text{CP}}^\nu = -\frac{\lambda \sin \phi}{12\sqrt{2}}, \quad (23)$$

where  $\phi$  is one of the Majorana phases. If  $\sin \phi$  is near one, this contribution would be dominant over the one in Eq. (22) and would give a maximum  $J_{\text{CP}}^\nu$  as large as 0.014, which corresponds to  $\delta \approx 45^\circ$ . We expect that, irrespective of the nature of the neutrino, future experiments have to measure a value of  $\delta$  between the two limits given by Eq. (22) and (23), i.e.,  $3^\circ < \delta < 45^\circ$ .

#### 4. Can the QLC relation arise from charged lepton mixing?

The presence of the Cabibbo angle in the MNSP matrix, as recent measurements of the solar mixing angle indicates, at first sight may suggest that all deviations from the exact bimaximal ansatz may be a contamination coming from the charged lepton mixing matrix. We have seen that the patterns in the fermion spectra suggest that there is a leptonic basis where the electron mass is generated from the mixing between the first two flavor families. This basis is most probably the gauge flavor basis of the theory, where quarks and leptons unify in common representations. It is precisely in this basis where one would expect the neutrino mixing matrix to be exactly bimaximal. Nevertheless, if this was the case we would obtain that  $\theta_{12} = \frac{\pi}{4} + \frac{\theta_C}{6}$  instead of the experimentally observed  $\theta_{12} = \frac{\pi}{4} - \theta_C$ , too small and of the opposite sign required to account for the QLC relation. If one insists to fully generate the observed deviation from bimaximality in the MNSP matrix from the charged lepton mixing, the required mixing would be very large and as a consequence in such a basis the charged lepton mass matrix would adopt a very unnatural form in order to reproduce the correct electron mass [28]. Therefore, we believe that most probably the Cabibbo angle is already present in the neutrino mass matrix, or in other words the QLC relation must arise from the mechanism that generates the neutrino mass matrix and not from the charged lepton mixing. Of course, it is entirely possible that the QLC relation is only approxi-

<sup>2</sup> The observation that a hierarchical structure in the charged lepton mixing matrix would suppress a possible contribution to  $J_{\text{CP}}^\nu$  has been simultaneously made by Ref. [29].

mate and furthermore is accidental and a red herring and does not therefore need any explanation.

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